LITERATURE CITED

1. Yu. A. Buevich, "Instability in a self-modeling phase-transition front," Inzh.-Fiz. Zh., 40, No. 5, 818-827 (1981).
2. W. Mullins and R. Sekerka, "Stability of a planar phase interface in crystallization of a dilute binary alloy," in: Crystal-Growth Problems [Russian translation], Mir, Moscow (1968), pp. 106-126.
3. J. Cutler and W. Tiller, "Allowance for particle-attachment kinetics to a crystal in analyzing the stability of a cylinder crystallizing from a binary melt," in: Crystal-Growth Problems [Russian translation], Mir, Moscow (1968), pp. 176-196.
4. W. Mullins and R. Sekerka, "The morphological stability of a particle growing by diffusion or heat loss," in: Crystal-Growth Problems [Russian translation], Mir, Moscow (1968), pp. 89-105.
5. Yu. A. Bychkov and S. V. Iordanskii, "Instability of a phase boundary during a phase transformation," Zh. Prikl. Mekh. Tekh. Fiz., No. 5, 45-51 (1980).
6. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford Univ. Press (1959).
7. L. I. Rubinshtein, Stefan's Problem [in Russian], Zinatne, Riga (1967).
8. I. K. Kikoin (ed.), Tables of Physical Quantities (Handbook) [in Russian], Atomizdat, Moscow (1976).
9. B. Ya. Lyubov, The Theory of Crystallization in Large Volumes [in Russian], Nauka, Moscow (1975).

THERMOELASTIC STABILITY OF COOLED LASER MIRRORS

V. V. Kharitonov and S. B. Koshelev

UDC 539.3:621.375.826

The article examines the interconnection between permissible thermal stresses, deformations, and thermal loads on laser mirrors fixed by different systems.

Installations based on powerful lasers contain on the path of the light a large number of elements, especially mirrors. On each of them the incident wave front is being distorted. Accumulation of distortions on different elements leads to defocusing of the beam and makes it unsuitable for practical purposes. The quality of the mirrors, as one of the causes of distortion of the wave front in multielement systems, must therefore satisfy particularly stringent requirements. Normal deformations must not exceed 1/10-1/40 of the wavelength of the laser beam [1]. In addition, the transverse temperature gradient in the mirror, which is proportional to the absorbed thermal flux, may cause impermissible stresses in it. Thus the permissible luminous loads on laser mirrors are limited by the permissible thermal strains and stresses. The present article shows how the condition of mounting a plane mirror and the intensity of cooling affect the thermal stresses and strains in the mirror and the permissible luminous load imposed on the mirror.

The simplest form of a laser mirror is a plane disk with constant thickness δ and radius R. One surface of the disk is illuminated (heated), and the other surface is cooled by a heat carrier with constant heat transfer coefficient α. The intensity of the irradiation is uniform over its entire surface, i.e., it does not depend on the radial coordinate. (The case with nonuniform illumination requires a special analysis.)

We will first examine two limit cases of mounting mirrors: freely supported by a rigid base and rigidly secured on its circumference.

When a mirror is heated by a laser pulse, the pulse duration t is such that the thickness $\sqrt{at}$ of the heated layer (within the time that the pulse acts) is much smaller than the thickness δ of the mirror; the temperature field in it T(x, t) is correlated with the pulse energy I (J) by the equation of thermal balance

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 4, pp. 640-646, October, 1983. Original article submitted June 14, 1982.

0022-0841/83/4504-1176$07.50

$$AI = \pi R^2 \int_0^\delta \rho c_p T(x, t)\, dx. \quad (1)$$

In consequence of the thermal expansion of the surface layer, the mirror endeavors to bend toward the incident luminous flux. If it is not secured along the circumference, then the normal deformation at the center in its bending is correlated with the temperature by the expression [2]

$$\omega = \frac{6\beta R^2}{\delta^3} \int_0^\delta T(x, t)\left(\frac{\delta}{2} - x\right) dx. \quad (2)$$

This expression was obtained in the quasistatic approximation of the theory of thermoelasticity, and it is therefore suitable when the duration of the laser pulse is much greater than the time δ/c necessary for the sound wave to propagate from the heated to the cooled mirror surface. For instance, with $\delta = 4$ mm, $c = 4$ km/sec, we must have $t \gg 10^{-6}$ sec.

If we limit in (2) the deflection at the end of the pulse effect by the maximum permissible value of $[\omega]$ and take into account that $\sqrt{at} \ll \delta$, we obtain from (1) and (2) the restriction for the pulse energy:

$$I = \frac{\pi}{3} \frac{\rho c_p}{A\beta} \delta^2 [\omega]. \quad (3)$$

An analogous expression was obtained in [1]. It follows from (3) that the maximum permissible pulse energy does not depend on the pulse duration (when $t \gg \delta/c$), the area of the mirror, and the intensity of its cooling, but it increases rapidly with increasing thickness of the mirror. Since $[\omega] \approx \lambda/10$-$\lambda/40$, the luminous load on the mirror with shortwave lasers is bound to be minimal. Among the metals that reflect light well, copper, tungsten, and molybdenum have the highest value of the complex $\rho c_p/\beta$. It follows from (3), e.g., that with a pulse energy of 10^5 J a copper mirror has to be at least 80 mm thick if $A = 2\%$ and $[\omega] = 1$ µm (CO_2 laser).

In case of stationary illumination of the mirror with uniform density of the thermal flux $q = AQ/(\pi R^2)$, a linear temperature field $T_c + q\,(1/\alpha + (\delta - x)/k)$ forms in it, where T_c is the temperature of the heat carrier. According to the theory of thermoelasticity for plates [2], when the temperature field is linear over the thickness of the mirror, there are no stresses in the mirror, and it is being bent on account of the thermal expansion of the heated surface. If we substitute the linear temperature field into formula (2) and confine bending at the center to the permissible value $[\omega]$, we find the maximum permissible luminous load on the mirror (W):

$$Q = \frac{q\pi R^2}{A} = 2\pi \frac{k[\omega]}{A\beta}. \quad (4)$$

Hence it follows that the maximum thermal power absorbed by a freely supported mirror does not depend either on the thickness or on the dimensions of the mirror, but only on its thermophysical properties and the permissible deformation. It follows from Table 1 that with $[\omega] = 1$ µm the permissible thermal power amounts all in all to 63-180 W, and luminous power in reflection of 99% amounts to 6.3-18 kW. When radiation with shorter waves is used, this power will be even lower.

TABLE 1. Some Limit Parameters of Laser Mirrors Calculated by Formulas (4)-(6), (8) for $[\omega] = 1$ µm, $Bi = \infty$, $\nu = 1/3$

Metal	AQ (W) by (4)	$[\Delta T]$ (°K)	$q\delta$ (kW/m) by (6)	$100\,\delta/R$ by (8) for h = 1
Aluminum	63	15—25	4—6	1,9—2,5
Copper	150	13—25	5—10	1,0—1,7
Tungsten	180	40—100	5—13	1,1—1,8
Invar	70	—	—	—

In the other limit case, when the mirror is rigidly secured on its circumference in such a way that deformations are impossible either on the circumference, radially, or in the normal directions, then according to the theory of thermoelasticity [2], there are no deformations at all, $\omega(r) = 0$ with uniform illumination, but thermal compressive stresses arise in the mirror; these stresses are proportional to $\beta E \Delta T/(1-\nu)$, where ΔT is the overheating of the mirror surface relative to the temperature of the unstressed state. Since the stresses must not exceed a maximum permissible value $[\sigma]$ (e.g., the yield strength), there follows from this a limitation on overheating of the mirror $[\Delta T] < (1-\nu)[\sigma]/(\beta E)$ and on the luminous load in the pulsed regime

$$\frac{I}{\pi R^2} = \frac{\sqrt{\pi a t}}{2A} \frac{(1-\nu)\rho c_p [\sigma]}{\beta E} \tag{5}$$

and in the steady-state regime

$$\frac{Q}{\pi R^2} = \frac{(1-\nu) k [\sigma]}{A \beta E} \frac{\mathrm{Bi}}{1+\mathrm{Bi}} \frac{1}{\delta}. \tag{6}$$

Here it is indispensable to point out that the maximum permissible stresses for the steady-state and the pulsed operating regimes of the mirrors may differ in magnitude. Usually, the shorter the pulse duration, the larger the permissible stress. However, the question of selecting the maximum permissible stress in connection with laser mirrors requires a special investigation.

It follows from (5) that the threshold of impulse rupture of a mirror is proportional to the square root of the pulse duration; is maximal for copper mirrors and is equal to 5-10 kJ/m^2 for $t = 10^{-9}$ sec and $A = 1\%$. Hence it follows in particular that for laser thermonuclear reactors [3] not less than a 100-m^2 surface of focusing mirrors with $I = 10^6$ J is required.

It can be seen from (6) that the product of the steady-state density of the thermal flux in the mirror and the thickness of the mirror ($q\delta$) depends only on the properties of the material of the mirror (with intense cooling, when $\mathrm{Bi} > 1$), it amounts to about 10 kW/m (Table 1), and the smaller the thickness of the mirror, the larger its thermal and permissible luminous loads. For instance, a copper mirror 2 mm thick withstands a thermal flux of up to 5-10 MW/m^2. However, when the mirror is thinner, a number of new limitations arise. Firstly, removal of large heat fluxes by the heat carrier may prove to be impossible (e.g., in conseqence of crisis of boiling of the heat carrier). Secondly, thin mirrors are less resistant to transverse deformations. We will examine this last question in greater detail. A plate, rigidly fixed or pin-supported on its circumference, does not become deformed in the absence of transverse forces as long as the compressive stresses in its plane do not exceed some critical value [2]

$$\sigma_{cr} = \frac{\varkappa^2}{12} \frac{E}{1-\nu^2} \left(\frac{\delta}{R}\right)^2, \tag{7}$$

where $\varkappa = 4$ in the case of rigid fixing [4], and in case of pin support the value of the coefficient $\varkappa$ is found from the solution of the transcendental equation [5] $J_0(\varkappa) = (1-\nu)/\varkappa \cdot J_1(\varkappa)$. Thus, for $\nu = 1/3$ we obtain $\varkappa \approx 2$. Therefore, for a pin-supported plate the critical stress (of loss of stability) is approximately four times smaller than for a rigidly mounted plate.

With $\sigma > \sigma_{cr}$, fixed plates are unstable and bend. The critical stress (7) in its turn must not exceed some maximum permissible value bounding the range of elastic deformations, Taking into account that the plane of the mirror is not ideal, we limit the critical stress (7) by the value $n[\sigma]$, where $n = 2$-4 is the safety factor for instability. In that case we obtain from (7) the condition for the thickness and dimension of a mirror that is thermoelastically stable:

$$\frac{\delta}{R} \geqslant \sqrt{\frac{3}{4}(1-\nu^2)\frac{[\sigma] n}{E}}. \tag{8}$$

It follows from Table 1 that with $n = 1$ a circular mirror is stable when $\delta/R > 0.01$-0.02. With a view to the margin $n = 4$, its relative thickness δ/R doubles. The expressions (6) and

(8) obtained here, which limit the permissible dimensions and thermal fluxes to circular mirrors, enable us to determine the permissible power (W) that is absorbed by a mirror rigidly fixed on its circumference:

$$Q = q\pi R^2 = \frac{4\pi}{3(1+\nu)n}\,\frac{k\delta}{\beta}\,\frac{\mathrm{Bi}}{1+\mathrm{Bi}}. \tag{9}$$

If we compare expressions (9) and (4), we may conclude that a fixed mirror may, in the range of thermoelastic stability, withstand thermal loads approximately $\delta/[\omega]$ times larger than a mirror not fixed on its circumference (with good cooling, when Bi > 1). For instance, with $[\omega] = 1$ μm and $\delta = 1$ mm, the thermal load (9) is almost 1000 times larger than Q from the first column of Table 1, attaining values of the order of 100 kW.

When transverse forces act, e.g., due to the pressure difference ΔP in the heat carrier and in the medium surrounding the mirror, a fixed mirror is absolutely unstable. When transverse and radial stresses act jointly, maximum deflection (normal displacement) at the center can be determined from the approximate equation [4]

$$\left(\frac{\omega}{\delta}\right)^3 + 3B\left(\frac{\omega}{\delta}\right) = 2C, \tag{10}$$

where

$$3B = \frac{56}{23-9\nu}\left(1-\frac{\sigma}{\sigma_{cr}}\right);\quad 2C = \frac{21(1-\nu)}{2(23-9\nu)}\,\frac{\Delta P}{E}\left(\frac{R}{\delta}\right)^4.$$

It follows from Eq. (10) that when the radial thermal stresses are small ($\sigma/\sigma_{cr} \ll 1$) and the deformations are small ($\omega/\delta \ll 1$), the first term on the left-hand side may be neglected. Therefore, the displacement of the center of the mirror, as in the isothermal case, is proportional to the pressure difference ΔP:

$$\frac{\omega}{\delta} = \frac{3(1-\nu)}{16}\,\frac{\Delta P}{E}\left(\frac{R}{\delta}\right)^4. \tag{11}$$

However, when the thermal stresses in the plane of the mirror are close to critical (7), i.e., the coefficient $B \approx 0$ in Eq. (10), then the deformation of the mirror increases in accordance with the expression

$$\frac{\omega}{\delta} = \left[\frac{21(1-\nu)}{2(23-9\nu)}\,\frac{\Delta P}{E}\left(\frac{R}{\delta}\right)^4\right]^{1/3}. \tag{12}$$

In the general case, for $D = C^2 + B^3 > 0$, i.e., when the radial stresses do not exceed the critical ones ($\sigma < \sigma_{cr}$), the solution of Eq. (10) has the form

$$\frac{\omega}{\delta} = (\sqrt{D}+C)^{1/3} - (\sqrt{D}-C)^{1/3}. \tag{13}$$

The effect of the transverse pressure ΔP and of the radial stresses σ/σ_{cr} on the magnitude of the deformation of the center of the mirror can be seen in Fig. 1. It follows from this figure that when there are radial stresses, the deflection of the mirror is one or two orders of magnitude larger than deflection due only to transverse pressure.

To reduce radial thermal stresses (with specified thermal flux), it is indispensable to reduce the thickness of the reflecting layer, because thereby the temperature gradient across the thickness of the mirror is reduced, and to reduce deflections under the effect of transverse pressure, rigid supports in the form of ribs, tongues, etc., may be used, which is equivalent to reducing the radius of the mirror. In the limit, when the thickness of the reflecting layer and of the supports holding it are small, we arrive at the idea of cooled mirrors with porous base through which the heat carrier is filtered at great speed [6]. However, porous structures are characterized by high hydraulic resistance, in consequence of which a large pressure gradient ΔP between the inlet and outlet collectors of the heat carrier may arise when the mirror is intensively cooled. As a result, sections of the reflecting layer between supports of the body may bend, and on account of that some

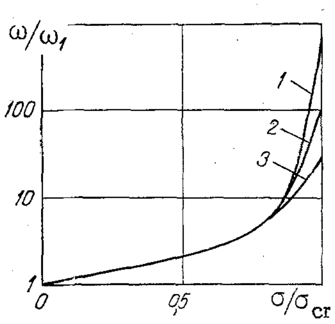

Fig. 1. Effect of radial stresses and of transverse pressure on the normal deformation of the center of a copper mirror. Calculation by formula (13); ω_1 calculated by formula (11) for $\omega_1/\delta = 10^{-4}$ (1), 10^{-3} (2), 10^{-2} (3).

effective "microroughness" of the mirror surface may be produced. Since the height of the "microroughness" must not exceed the specified value $[\omega]$, there arises a limitation on the pressure gradient in the heat carrier and on the geometric dimensions of the porous body. We will estimate ΔP for the limit conditions of fixing the reflecting layer to the porous body by a system of parallel bands (when the body is formed by stiffening ribs) and by a system of spots (when the body has a brushlike structure or is made of sintered powders, rings, etc.).

The permissible pressure gradient ΔP is determined from the solution of the equation of a thin plate [2] $\nabla^4\omega = \Delta P/D$, where $D = E\delta^3/12\,(1-\nu^2)$ is the stiffness of the reflecting layer. Assuming that the layer is rigidly fixed at the places of contact with the body, we find ΔP for band contact:

$$\Delta P = 384\,\frac{D\,[\omega]}{b^4}. \tag{14}$$

In the case of contact according to the system of regularly spaced circular spots with diameter d and pitch s we have

$$\Delta P = 1024\,\frac{1+\nu}{5+\nu}\,\frac{D\,[\omega]}{s^4}\,\frac{1}{\Psi(\varepsilon)}, \tag{15}$$

where

$$\Psi(\varepsilon) = \frac{1+\nu}{5+\nu}\left\{(1-\varepsilon)\left(\varepsilon + \frac{5+3\nu}{1-\nu}\right) + 2\,\frac{3+\nu}{1-\nu}\ln\varepsilon + \right.$$
$$\left. + 2\,\frac{\left[2(1-\varepsilon)\,\frac{\nu}{1-\nu} + \left(\frac{1+\nu}{1-\nu} - \varepsilon\right)\ln\varepsilon\right]\left[\varepsilon^2 + 2\varepsilon\,\frac{1+\nu}{1-\nu}\,\frac{3+\nu}{1-\nu}\right]}{\frac{1-\varepsilon}{1-\nu} - \varepsilon\ln\varepsilon}\right\}$$

is a function of the relative contact area $\varepsilon = (d/s)^2$ of the reflecting layer with the body and of the Poisson ratio. The function $\Psi(\varepsilon)$ changes from $\Psi = 1$ for $\varepsilon = 0$ to $\Psi = 0$ for $\varepsilon = 1$. Obviously, with $\varepsilon = 0$ (point contact) the deformation of the layer is maximal with the specified pressure, and with the specified deformation the pressure is minimal.

The permissible filtration rates of the heat carrier are presented in Fig. 2. The calculations were carried out by the formula

$$\Delta P = \xi\,\frac{\rho W^2}{2}\,\frac{L}{d_h}. \tag{16}$$

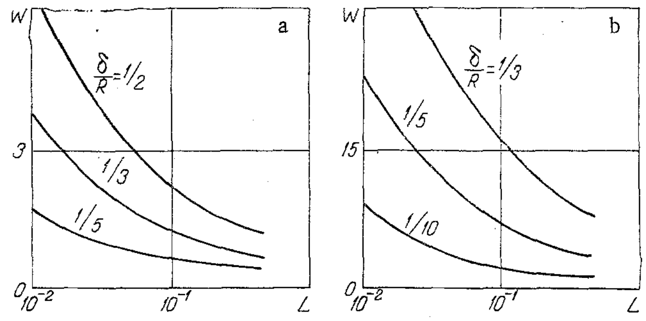

Fig. 2. Permissible filtration rates of water in the porous body of a copper mirror in dependence on the distance between collectors and the structure of the body: a) powdered; b) brushlike. Calculation by formulas (15), (16) for $\Psi = 1$, $[\omega] = 0.1$ μm.

The heat carrier was water. The friction coefficients were the following: for the brushlike structure $\xi = 0.1$, $d_h = d$, where d is the tongue diameter; for sintered powders $\xi = 13$-36, $d_h = d$, where d is the diameter of the powder particles.

It follows from Fig. 2 that with increasing distance L between the collectors, the speed W greatly decreases. The most stringent are the limitations imposed on the permissible filtration rate in a mirror body made of sintered powders. This is only natural, because such structures have the highest hydraulic resistance. In a body formed by a system of parallel ribs there are practically no limitations of ΔP on the filtration rate (if cavitation is not taken into account).

NOTATION

σ, $[\sigma]$, σ_{cr}, stress, maximum permissible, and critical stress, respectively, in the mirror; $\varkappa$, numerical coefficient in (7); E, Young's modulus; ν, Poisson ratio; ω, $[\omega]$, deformation and permissible deformation, respectively, of the mirror; δ, thickness of mirror; R, radius of mirror; d_h, hydraulic diameter; b, width of the slit between ribs; L, distance between collectors; W, filtration rate of heat carrier; ξ, coefficient of hydraulic resistance; ΔP, pressure gradient; J_0, J_1, Bessel function; λ, wavelength of radiation; x, coordinate in direction of depth of mirror; c, speed of sound; t, pulse duration; α, heat transfer coefficient; a, thermal diffusivity; ρ, density; c_p, heat capacity; A, absorption coefficient; β, coefficient of thermal expansion; q, heat flux density; Q, luminous load; I, pulse energy; k, thermal conductivity; Bi, Biot numer.

LITERATURE CITED

1. H. E. Bennet, "Thermal distortion thresholds for optical trains handling high pulse powers," in: Laser Induced Damage in Optical Materials, U. S. Dep. Commer. Nat. Bur. Stand. Spec. Publ. (1976), pp. 11-24.
2. B. A. Boley and J. H. Weiner, Theory of Thermal Stresses, Wiley (1960).
3. L. S. Kokorev and V. V. Kharitonov, in: Direct Transformation of Energy and Thermonuclear Power Installations, V. I. Subbotin (ed.), Atomizdat, Moscow (1980), pp. 179-193.
4. I. I. Goldenblat, Calculation of Strength, Stability, and Vibrations under Conditions of High Temperatures [in Russian], Mashinostroenie, Moscow (1965).
5. B. G. Korenev, in: Problems of the Theory of Thermal Conductivity and Thermoelasticity [in Russian], Nauka, Moscow (1980), pp. 318-327.
6. V. V. Apollonov, P. I. Bystrov, V. F. Goncharov, et al., "Prospects of using porous structures for cooling elements of power optics," Kvantovaya Elektron., 6, No. 12, 2533-2545 (1979).